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AN EXAMPLE IN CONNECTION WITH THE JACOBIAN CONJECTURE

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Introduction. Let F and G be polynomials of two complex variables x and y. The well-known Jacobian conjecture consists in the following: if $F_xG_y - F_yG_x = \text{const} \neq 0$, then the polynomials F and G determine an invertible mapping $\mathbf{C}^2 \to \mathbf{C}^2$ (see, for example, [1]). In this case it is easy to show that the inverse mapping is also polynomial.

The Jacobian conjecture can be reformulated in the following way. Let ℓ be an infinitely remote direct line in $\mathbf{CP}^2$, U its tubular neighborhood (the exterior of a big ball in $\mathbf{C}^2$), and let f_1, f_2 be meromorphic functions on U, holomorphic on $U - \ell$ and defining a locally one-to-one mapping $f\colon U - l \to \mathbf{C}^2$. Then the Jacobian conjecture is equivalent to the injectivity of this mapping. Indeed, the functions f_1 and f_2 by the theorem on elimination of compact singularities can be continued to holomorphic functions on the whole $\mathbf{C}^2 = \mathbf{CP}^2 - l$. From the meromorphic property of these functions on $\mathbf{CP}^2$, and the holomorphic property on $\mathbf{C}^2$ it follows that they are polynomial, and from the fact that the Jacobian is not null outside some ball it follows that it is not null on the whole $\mathbf{C}^2$.

If with the help of the σ-processes we solve the undefined points of the mapping f, then the meromorphic property of the original mapping leads to rather strong limitations on the topology of the resulting surface and its mappings. Moreover, the image of the branching curve is an infinitely distant direct line, each with one branch at infinity.

Now we shall formulate the main result of the present note (Sec. 1 is devoted to its proof).

Proposition 0.1. There exists a smooth (noncompact) complex-analytic surface $\tilde{X}$, with a smooth curve $\tilde{L}$ in it, isomorphic to $\mathbf{CP}^1$, with self-intersection number +1, and two functions f_1, f_2, meromorphic on $\tilde{X}$, such that the mapping $\tilde{X} - \tilde{L}$, determined by these functions, $f\colon \tilde{X} - \tilde{L} \to \mathbf{C}^2$ is locally one-to-one, but not injective.

It is obvious that if $\tilde{U}$ is a tubular neighborhood curve $\tilde{L}$, and U is a tubular neighborhood of the direct line ℓ on $\mathbf{CP}^2$, then the pairs $(\tilde{U}, \tilde{L})$ and (U, ℓ) are C^∞-diffeomorphic. Thus there exists a smooth noninvertible embedding of the exterior of some ball into $\mathbf{C}^2$,

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which resembles a polynomial mapping with respect to its geometrical properties. If, however, the pairs $(\tilde{U}, \tilde{L})$ and (U, ℓ) were bi-holomorphic equivalent, then, as we mentioned above, we would obtain a counterexample to the Jacobian conjecture.

On the other hand, if the restriction of the mapping f to the boundary $\tilde{S}$ of the tubular neighborhood of the curve $\tilde{L}$ (clearly, $\tilde{S}$ is a three-dimensional sphere, and $f|_{\tilde{S}}$ is an embedding) could be continued to an embedding of the four-dimensional ball $B^4 \to C^2$, then, obviously, it would be possible to lift on $B^4 \cup_{\tilde{S}} \tilde{C}$ an analytic structure so as to obtain a smooth rational curve with self-intersection number +1, whose complement D would be homeomorphic to R^4. It has been shown in [2, p. 85] that in this case D would be isomorphic to C^2, hence again we would obtain a counterexample to the Jacobian conjecture.

It can be verified that the Gauss degree of the mapping of $\tilde{S}$ into the sphere, obtained from the embedding $f|_{\tilde{S}}$, is not one. Therefore the mapping constructed in this note cannot be extended to a mapping providing a counterexample to the Jacobian conjecture. The fact that this mapping is not extendible to a counterexample is also an immediate consequence of Lemma 4.2 in [3].

In Sec. 2 with the help of the Lodaira theorem [4] it has been shown that a smooth rational curve with a positive self-intersection number on a smooth analytic surface (in particular, the curve $\tilde{L}$ in the constructed example) has an arbitrarily small strongly pseudoconcave tubular neighborhood. For such a neighborhood we can take the union of curves corresponding to the points of a sufficiently small ball in the manifold of rational curves on the given surface. This fact, obviously, is interesting for its own.

1. Construction of an Example

1.1. Construction of the Image of a Branching Curve. Let us consider a curve K in C^2, parametrized with the polynomials

$$P(t) = t^{21} + 6t^{14} + \frac{63}{2}t^{12} + \frac{63}{4}t^{11} - \frac{63}{2}t^{10} - \frac{63}{2}t^{9} - \frac{39}{2}t^{7} +$$

$$+ 63t^5 + \frac{63}{2}t^4 + \frac{819}{8}t^3 + \frac{819}{8}t^2 - \frac{9261}{32}t - \frac{4409}{8},$$

$$Q(t) = t^{14} + 4t^7 + 21t^5 - \frac{21}{2}t^4 - 21t^3 - 21t^2 - 17.$$

These polynomials satisfy the relations

$$\deg_t \left(P(t)^2 - Q(t)^3 + \frac{9261}{16}Q(t)\right) = 13. \qquad (1)$$

$$\text{g.c.d.}\,(P', Q') = 1. \qquad (2)$$

Let X be a compactification of $\mathbf{C}^2$ in which curve K has no singularities at infinity, i.e., X is a smooth compact algebraic surface containing a (reducible) curve L, such that $X - L \cong \mathbf{C}^2$, and the closure k of curve K transversally meets curve L at a unique point p_∞, which is a smooth point both on k and L.

Pair (Y, A), where A is a curve on a smooth complex-analytic surface Y, will be called regular if all irreducible components of curve A are smooth compact rational curves, meeting transversally but at most pairwise. We shall associate with each regular pair a dual weighted graph. The vertices of this graph correspond to irreducible components, and its edges to the points of intersections of the corresponding components. The weight of a vertex is minus the self-intersection number of the corresponding component. The signs have been so chosen in order that the determinant of the intersection matrix be not changed by blowing up a point on this curve. If it is clear which surface is concerned then we shall speak about a dual graph of a curve.

Let Γ be a dual weighted graph of pair (X, L). By (1) it has been constructed as of Fig. 1 (the edge with an arrow corresponds to point p_∞).

1.2. General Description of the Example. We shall construct a regular pair $(\tilde{X}, \tilde{L})$ with a dual graph displacyed on Fig. 2, and a holomorphic mapping f: $\tilde{X} \to X$, such that the restriction of f to $\tilde{X} - \tilde{L}$ is locally one-to-one, $f(\tilde{X} - \tilde{L}) \subset X - L$ and $f(\tilde{L}) \subset L \cup k$. This will be the proof of Proposition 0.1. Indeed, if some irreducible component of the curve $\tilde{L}$ has self-intersection number −1, and does not intersect more than two other components of the curve $\tilde{L}$, then it can be contracted to a point, and then we obtain again a regular pair. In order to obtain its dual graph we should proceed with $\tilde{\Gamma}$ as follows: choose an arbitrary vertex v with weight 1, with not more than two edges coming out of it; move away the vertex v and the edges coming out of it; if from vertex v two edges vu_1 and vu_2 were coming out, then add the edge u_1u_2; reduce the weights of vertices next to v by one.

It is easy to verify that continuing the above procedure until the vertices with weight 1 are exhausted, after 228 contractions from graph Γ we obtain a linear graph with weights (2, 2, 2, 2, 2, 2, 0, −1). This graph, by means of blowing up and contractions, can be converted into a graph consisting of a single vertex with weight −1 (cf., [2, Lemma 5], it can be also easily verified directly). Obviously, the resulting regular pair satisfies the hypothesis of Proposition 0.1.

To construct $\tilde{X}$ and f we shall choose around every irreducible component ℓ of the curve L a tubular neighborhood $U(\ell)$, such that $U(\ell)$ meets $U(\ell')$ only if ℓ meets ℓ', and in this case the intersection will be bicylinder with the center $l \cap l'$. Let ν: $k^* \to k$ be the normalization of the curve k. By (2) ν is an embedding. Therefore there exists a tubular neighborhood U^* of the null section of some linear bundle $E^* \to k^*$ and an embedding ν^*: $U^* \to X$ such that $\nu^*|_{k^*} = \nu$ (we shall identify k^* with the null section in E^*). Let $U(k) = \nu^*(U^*)$ and $U = \bigcup_{l<L} U(l)$ ($l < L$ means: ℓ is an irreducible component of curve L). For a subgraph Γ_i of graph Γ (see Fig. 1) we put $U(\Gamma_i) = \bigcup_{l<L_i} U(l)$, where by L_i we denote the union of irreducible components corresponding to the vertices of graph Γ_i. Let ℓ_1, ℓ_2 be curves corresponding to the vertices v_1, v_2 (see Fig. 1).

We shall construct the surface $\tilde{X}$ and mapping f: $\tilde{X} \to X$ separately over each set U(k), $U(\ell_1)$, $U(\ell_2)$, $U(\Gamma_1),\ldots,U(\Gamma_4)$, and further we shall show the congruence on the intersections. Moreover, over set U mapping f will be a branched covering with the branching over $L \cup k$.

1.3. Construction of Coverings Over $U(\Gamma_i)$. Let $A = (a_{ij})$ be an integer 2 × n-matrix such that det(A(i, j)) > 0 for i < j [by A(I, j) we denote the 2 × 2 submatrix of matrix A composed with the i-th and j-th columns]. Denote by $\Sigma(A)$ the fan in $\mathbf{R}^2$, whose one-dimensional cones are generated by the columns of the matrix A, and the two-dimensional ones by the pairs of adjacent columns. By X(A) we denote the corresponding two-dimensional toroid manifold. It is smooth if all det(A(i, i + 1)) = 1 (cf., [5]).

LEMMA 1.3.1. Let $A = (a_{ij})$ be an integer 2 × n matrix, such that det(A(i, i + 1)) = 1 for i = 1,...,n = 1, and let ℓ be the closure of a one-dimensional orbit of the manifold X(A) corresponding to the i-th column. Then, if 1 < i < n, then ℓ is compact and its self-intersection number is equal to −det(A(i − 1, i + 1)).

With the help of Lemma 1.3.1 it is easy to construct a matrix A, such that the dual graph of the union of compact orbits of the manifold X(A) is isomorphic to the given linear weighted graph. Let A_i (i = 1,...,4) be such matrices for the graphs Γ_i and $X_i = X(A_i)$. By the tautness of the toroid singularities [6] it is easy to show that L_i can be identified with the union of the compact orbits of the manifold $X(A_i)$, and $U(\Gamma_i)$ with its neighborhood.

For each i = 1,...,4 we shall consider a linear mapping B_i: $\mathbf{R}^2 \to \mathbf{R}^2$ given by a matrix composed with the first and the last column of the matrix A_i, and let $\tilde{\Sigma}_i$ be a canonical partition of the fan $B_i^{-1}(\Sigma_i)$ into primitive (i.e., generated by the bases of the integer lattice-cones (cf., [5, 8.4]). It can be directly verified that if σ is one of two extreme two-dimensional cones of the fan Σ_i (i = 1,...,4), then $B_i^{-1}(\sigma)$ is a primitive cone. From this fact, and also from the coordinate description of toroid manifolds and mappings [7] there follows

LEMMA 1.3.2. Let p be a null-dimensional orbit in $X(\Sigma_i)$, corresponding to one of the extreme cones of the fan, Y_i the union of compact one-dimensional orbits, and β_i a toroid mapping associated with B_i. Then $\beta_i|_{\beta_i^{-1}(V)}$ for some neighborhood V of the point p is a cyclic covering of degree det B_i, branching over $V \cap Y$.

Let $\tilde{X}_1 = 12X(\tilde{\Sigma}_1)$, $\tilde{X}_2 = 18X(\tilde{\Sigma}_2)$, $\tilde{X}_3 = X(\tilde{\Sigma}_3) \bigsqcup 7X_3$, $\tilde{X}_4 = 4X(\tilde{\Sigma}_2) \bigsqcup 8X_4$ (nX denotes a disjoint union of n copies of space X), and let f_i: $\tilde{X}_i \to X_i$ be a mapping whose restriction to the components are either identity mappings, or the mappings β_i. Put $\tilde{U}(\Gamma_i) = f_i^{-1}(U(\Gamma_i))$, $\tilde{L}_i = f_i^{-1}(L_i)$. A direct computation shows that the following lemma holds.

LEMMA 1.3.3. The dual graph of the pair $(\tilde{X}_i, \tilde{L}_i)$ is isomorphic (up to the weights) to graph $\tilde{\Gamma}_i$ (see Fig. 2: where some vertices and subgraphs have been indicated; the subgraph generated by all the remaining ones is $-\tilde{\Gamma}_4$).

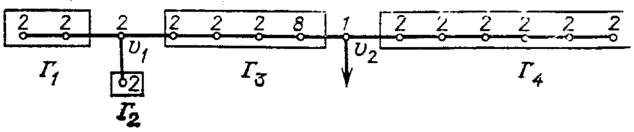

Fig. 1

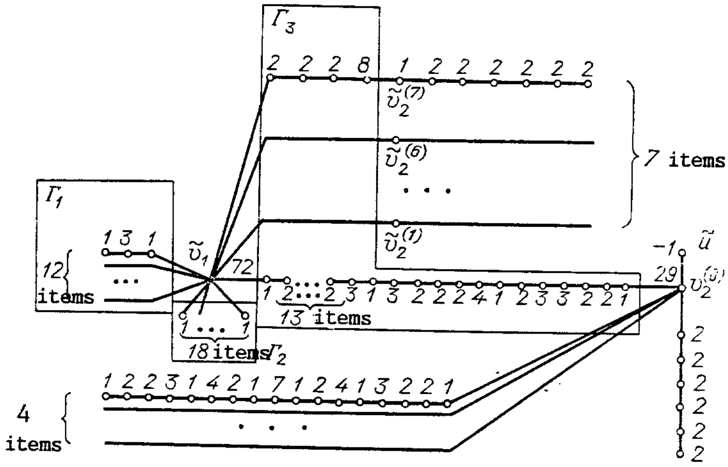

Fig. 2

1.4. Construction of Coverings Over $U(\ell_i)$. Let $i = 1$ or 2, $D_i = U(l_i) \cap ((L \cup k) - l_i)$ This union of three-two-dimensional discs transversally intersects ℓ_i. Therefore $U(\ell_i) - D_i$ is homotopically equivalent to a two-dimensional sphere with three punctures; m-fold coverings of the set $U(\ell_i)$ (necessarily connected), branching over D_i, are given by the homomorphism of the fundamental group $G_i = \pi_1(U(l_i) - D_i)$ into the group S(m) of permutations of m elements.

We shall now denote by a_1, b_1, c_1 the generators of group G_1, corresponding to traveling around L_1, L_2, L_3, and by a_2, b_2, c_2 the generators of group G_2 corresponding to traveling around L_3, L_4, k. These elements satisfy the unique relationships $a_ib_ic_i = 1$.

Let $\varphi_i\colon G_i \to S(36)$ be homomorphisms, such that

$$\begin{aligned}
\varphi_1(a_1) = {} & (2\ 3\ 30)(5\ 6\ 31)(10\ 11\ 32)(13\ 14\ 33)(18\ 19\ 34)\\
& (22\ 23\ 35)(25\ 26\ 36)(1\ 7\ 4)(9\ 15\ 12)\ (17\ 28\ 20)\\
& (21\ 27\ 24)(8\ 29\ 16),\\
\varphi_1(b_1) = {} & (2\ 4)(5\ 7)(10\ 12)(13\ 15)(18\ 20)(22\ 24)(25\ 27)\ (3\ 30)\\
& (6\ 31)(11\ 32)(14\ 33)(19\ 34)(23\ 35)(26\ 36)\ (1\ 8)(9\ 16)\\
& (17\ 29)(21\ 28),\\
\varphi_1(c_1) = {} & \varphi_2(a_2) = (1\ 2\ 3 \ldots 28\ 29),\\
\varphi_2(b_2) = {} & (28\ 27 \ldots 22)\ (21 \ldots 15)\ (14 \ldots 8)\ (7 \ldots 1),\\
\varphi_2(c_2) = {} & (29\ 22\ 15\ 8\ 1),\\
\varphi_i(a_i)\,\varphi_i(b_i)\,\varphi_i(c_i) = {} & 1^{\dagger}.
\end{aligned}$$

Consequently, these homomorphisms are well defined. Let $g_i\colon \tilde U(l_i) \to U(l_i)$ be a 36-fold covering, branching over D_i, corresponding to homomorphism φ_i $(i = 1, 2)$. The curve $g_1^{-1}(l_1)$ is connected (on Fig. 2 vertex $\tilde v$ corresponds to it), and a $g_2^{-1}(l_2)$ splits into 8 disjoint curves (vertices $\tilde v_2^{(i)}$). On the curve corresponding to $\tilde v_2^{(0)}$ (and in its neighborhood) the mapping g_2 is 29-fold, and in neighborhoods of other curves it is one-fold.

1.5. Construction of a Mapping Over U(k). LEMMA 1.5. $k^*\cdot k^* = 5$.

COROLLARY. There exists a cyclic fivefold covering $\tau\colon \tilde U(k) \to U^*$, branching over k^*.

Let $h = \nu^*\tau$, $\tilde k = \tau^{-1}(k^*) = h^{-1}(k)$ (on Fig. 2 to curve $\tilde k$ corresponds vertex $\tilde u$).

Proof of Lemma 1.5. Irreducible components of curve L form a basis in Pic X. Since the intersection numbers of the basic elements with each other and with the divisor of k

†We write a product of permutations as a composition of mappings from the right to left, i.e., $(12)\cdot(23) = (123)$.

are known, we can find the decomposition of the divisor of k with respect to this basis and its self-intersection number: $k \cdot k = 91$. Having written the addition formula for each element of the basis we obtain a system of linear equations, from which we can find the decomposition of the canonical class of the surface X with respect to this basis. Further, we can compute the arithmetic type $\pi_a(k) = 43$. By (2) all singular points of the curve k are simple double. It is known that the arithmetic type of such curves is bigger than the normalization type by the number of the double points. Consequently, $\pi_a(k) = d$, where d is the number of the double points of curve k. It remains to prove that $k \cdot k = k^* \cdot k^* + 2d$. Indeed, let us choose a C^∞-section of the bundle E^*, close to the null section, whose image k_1^* is in U^* and which meets transversally k^* (recall that k^* is n image of a null section of E^*) at points which are not coimages of singular points under the mapping ν. Then $k \cdot k = \# k \cap \nu(k_1^*) = \# k^* \cap k_1^* + 2d = k^* \cdot k^* + 2d$. The lemma has been proved.

1.6. Gluing Together Constructed Solutions (Conclusion of the Proof of Proposition 0.1). The surface $\tilde{X}$ and the mapping $f: \tilde{X} \to X$ mentioned in Proposition 0.1 will be glued with the surfaces $\tilde{U}(\Gamma_i)$ (see 1.3), $\tilde{U}(\ell_i)$ (see 1.4), $\tilde{U}(k)$ (see 1.5), the mappings f_i, g_i, and h. From Lemma 1.3.2 it follows that the mappings f_i and g_i over the intersections of their give equivalent branching coverings. We shall join them together according to the mappings representing the equivalence. As a result we obtain a proper mapping $f_0: \tilde{U} \to U$ which is a covering over $U - (L \cup k)$, while its restriction to one of the components of the set $f_0^{-1}(U \cap U(k))$ is equivalent to the restriction of h to $h^{-1}(U \cap U(k))$. We shall glue $\tilde{U}$ with $\tilde{U}(k)$ according to this equivalence and denote by $\tilde{X}$ the resulting surface.

Denote by $\tilde{L}$ the curve made of $f_i^{-1}(L_i)$, $g_i^{-1}(l_i)$ and $h^{-1}(k)$. According to Sec. 1.2 it is enough to show that its dual graph is as on Fig. 2. For this, by virtue of Lemma 1.3.3 it remains to prove that $\tilde{l}_1 \cdot \tilde{l}_1 = -72, \tilde{l}_2^{(0)} \cdot \tilde{l}_2^{(0)} = -29$ and $k \cdot k = +1$. The first two equalities follow from part a) of the following lemma, and the third from part b) and Lemma 1.5.

LEMMA 1.6. Let $F: M_1 \to M_2$ be an m-fold branching covering of real four-dimensional (necessarily compact) manifolds, S_i a connected two-dimensional smooth compact submanifold in M_i, with S_1 0 $F^{-1}(S_2)$. Then

a) If F has a branching of order m along S_1, then $(S_1 \cdot S_1) = m(S_2 \cdot S_2)$.

b) If the image of the sub-manifold meets transversally S_2, then $m(S_1 \cdot S_1) = (S_2 \cdot S_2)$.

2. Existence of a Pseudoconcave Neighborhood

We shall prove the following theorem.

THEOREM 2.1. Let X be a smooth complex-analytic surface and Y a smooth compact rational curve with self-intersection number $m > 0$ lying on it. Then Y has an arbitrarily small strongly pseudoconcave tubular neighborhood.

LEMMA 2.2. Let $p: E \to \mathbf{CP}^1$ be a one-dimensional linear fiber bundle of degree $m > 0$, (x, y) a local analytic coordinates system in E, such that y is a linear coordinate in each fiber, x a coordinate on $\mathbf{CP}^1$, and $p(x, y) = x$. Let $y = u_j(x)$ $(j = 0,\dots,m)$ be some basis of holomorphic sections of the bundle E. Then the equality

$$\lambda (u_0(x), \dots, u_m(x)) = (u_0'(x), \dots, u_m'(x)) \tag{3}$$

is not satisfied at any point and for any λ.

Proof. Since the fiber bundle E is isomorphic to a standard one, in some coordinate system (x', y') the space of its holomorphic sections coincides with the space of polynomials in x' of degree not greater than m. Moreover, the coordinates x' and y' can be expressed by x, y by the formulas $x' = f(x)$, $y' = y/g(x)$ where the functions f' and g are nowhere null. Sections u_j in the coordinates (x', y') have the form $y' = P_k(x')$, where $P_0,\dots,P_m$ is a basis in the space of polynomials of degree not greater than m. Consequently,

$$u_k(x) = P_k(f(x))\, g(x). \tag{4}$$

Let us assume that at some point x and for some λ equality (3) holds. Substituting (4) into (3) we obtain

$$P_k(f(x))(\lambda g(x) - g'(x)) = P_k'(f(x))\, f'(x)\, g(x) \tag{5}$$

where $\{P_k\}$ is a basis in the polynomials space, therefore 1 and x' can be expressed as their linear combinations of its elements. Applying these linear combinations to the equalities (5) we conclude that the equalities $\lambda g - g' = 0$ and $f(\lambda g - g') = f'g$ hold at point x, which implies that f'g = 0. The obtained contradiction proves the lemma.

Proof of Theorem 2.1. Let N be a normal fiber bundle of the curve Y in the surface X, and let $\mathcal{O}(N)$ be the sheaf of germs of its holomorphic sections. Then $\mathcal{O}(N) \cong \mathcal{O}(m)$, consequently, $H^0(Y, \mathcal{O}(N)) \cong \mathbf{C}^{m+1}$, $H^1(Y, \mathcal{O}(N)) = 0$, and, by the Kodaira theorem [4], there exists on X a family of smooth curves Y_t, $t = (t_0, \ldots, t_m) \in B$ (B is a ball in $\mathbf{C}^{m+1}$ with the center at 0), satisfying the following conditions: (i) $Y_0 = Y$; (ii) $V = \{(x, t) \mid x \in Y_t\}$ is a smooth analytic hypersurface in X × B; (iii) the linear mapping $\sigma: T_0B \to H^0(Y, \mathcal{O}(N))$ is an isomorphism, where σ is defined in the following way. Let (x, y) be local coordinates on X, in which Y is given by the equation y = 0, and let (x, η) be the corresponding coordinates on N (i.e., η = dy). Since Y_t is a smooth curve, then by the implicit function theorem the surface V can be defined by the equation F(x, y, t) = 0, where F = f(x, t) − y. Then $\sigma(\partial/\partial t_i)$ is defined as a section of the bundle N, which in the coordinates (x, η) is given by the equation $\eta = \partial f/\partial t_i(x, 0)$. This definition does not depend on the choice of coordinates.

Let pr be the projection of X × B on X. On the set $X_0 = \mathrm{pr}(V)$ there is defined a real function $\varphi(p) = \min_{(p,t)\in V} |t|^2, p \in X_0, t \in B$. We shall show (which will conclude the proof of the theorem) that X_0 is a neighborhood (on X) of the curve Y, and that for sufficiently small ε > 0 the set $U_\varepsilon = \{p \in X_0 \mid \varphi(p) < \varepsilon\}$ is strongly pseudoconcave. It is sufficient to prove these two facts for every coordinate neighborhood on the surface X. We shall show it in the coordinates (x, y) introduced above.

Let $u_j(x) = \partial f/\partial t_j(x, 0)$. According to (iii) $\eta = u_j(x)$ (j = 0,...,m) are linearly independent sections of the sheaf $\mathcal{O}(N) \cong \mathcal{O}(m)$. From this we can easily derive that the functions u_j and u_j' cannot vanish anywhere all at the same time. Consequently,

$$|u|^2 > 0, \text{ where } u = (u_0, \ldots, u_m). \tag{6}$$

From (6) it follows that $\mathrm{rk}_{\mathbf{C}}(\mathrm{pr}|_V) = 2$ in some neighborhood of the curve Y, thus X_0 is a neighborhood of Y.

If a point $p = (x, y) \in X_0$ is fixed, then $\{t \in B \mid (p, t) \in V\}$ is a smooth submanifold in B, and $\min|t|^2$ is achieved at its internal point $t = (t_0,\ldots,t_m)$. Consequently, by the Lagrange theorem on the conditional extremum, there exists for this point an $\lambda \in \mathbf{C}$, such that

$$F = 0, \tag{7}$$

$$\lambda\, \partial F/\partial \bar{t}_j = \bar{t}_j, \quad j = 0, \ldots, m. \tag{8}$$

For $t_0 = \ldots = t_m = y = 0$ the Jacobian of the system (7), (8) with respect to the variables $\lambda, t_0,\ldots,t_m$ is equal to $\pm|u|^4$, and, according to (6), is not null. Consequently, for a sufficiently small y this system has a unique solution $\lambda = \lambda(x, y)$, $t_j = t_j(x, y)$, with $\varphi(x, y) = \Sigma\, |t_j(x, y)|^2$. Expanding function F = f(x, t) − y into a series with respect to t and inserting it into (7) we have

$$y = \sum_j u_j(x)\, t_j + O(|t|^2). \tag{9}$$

For every fixed j, expressing all t_i from (8) by means of t_j, and substituting them in (9), we obtain

$$y\bar{u}_j = |u|^2 t_j + O(|t|^2) \quad (j = 0, \ldots, m),$$

consequently ($|u|^2$ will be denoted by μ), $t_j(x, y) = y\bar{u}_j/\mu + O(|y|^2)$, and thus

$$\varphi(x, y) = y\bar{y}/\mu + O(|y|^3). \tag{10}$$

Let $\xi_1 = \varphi_y \mu/\bar{y} = 1 + O(|y|)$, $\xi_2 = -\varphi_x \mu/\bar{y} = y\mu_x/\mu + O(|y|)^2$. Clearly, $y \neq 0$ for $\varphi \neq 0$. Therefore, $\xi = (\xi_1, \xi_2)$ is a complex tangent vector to the surface $\varphi = \mathrm{const} \neq 0$. Differentiating (10) we shall find the values of the Levi form of the function φ on vector ξ:

$$L(\xi) = \sum_{i,j} \varphi_{x_i \bar{x}_j} \xi_i \bar{\xi}_j = -\frac{y\bar{y}}{\mu^3}(\mu\mu_{x\bar{x}} - |\mu_x|^2) + O(|y|^3) \quad \text{here } x_1 = x, x_2 = y$$

It remains to notice that by the Cauchy–Bunyakovskii inequality $\mu\mu_{x\bar{x}} - |\mu_x|^2 \geqslant 0$, where the equality is reached only when vector u' is proportional to vector u. However, the latter is impossible by virtue of Lemma 2.2. The theorem has been proved.

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A MINIMAL PERMUTATION REPRESENTATION OF THE FINITE SIMPLE JANKO GROUP J_4

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UDC 512

As shown in [1], the study of minimal permutation representations of known finite simple groups, i.e., representations on the cosets of proper subgroups of minimal index, is one of the paths to the classification of the finite simple groups. In the present paper we study a minimal permutation representation of the sporadic simple Janko group J_4.

It was shown by Mazurov and Mazurova [2] that a proper subgroup of minimal index in J_4 is a subgroup M isomorphic to a nontrivial split extension of an elementary Abelian group of order 2^{11} by the Mathieu group M_{24}. The index of M in J_4 is equal to 173067389.

THEOREM. The permutation representation of J_4 on the cosets of the subgroup $M = 2^{11}$: M_{24} has subdegrees *1*, 82575360, 28336, 15180, 32643072, 54405120, 3400320. The corresponding two-point stabilizers are isomorphic to $M = 2^{11} : M_{24}$, $L_2(23)$, $2^{1+12} : 3\Sigma_6$, $2^{3+12}(\Sigma_3 \times L_3(2))$, $[2^7]\cdot\Sigma_5$, $[2^8]\cdot(\Sigma_3 \times \Sigma_3)$, $[2^{12}](\Sigma_3 \times \Sigma_3)$.

Inour subgroup notation, 2^{m+k} denotes a special group of order 2^{m+k} with a center of order 2^m; 2^m denotes an elementary Abelian group of order 2^m; $[2^m]$ denotes a subgroup of order 2^m; A·B (A:B) denotes an extension (split extension) of A by B.

The permutation representation of a gorup $\mathcal{G}$ on the cosets of a subgorup H is the action of $\mathcal{G}$ on the set $\mathcal{G} \mid H = \{Hg \mid g \in \mathcal{G}\}$, defined by the rule

$$Hg \to Hgx \ \forall x \in \mathcal{G}.$$

The corresponding permutation character χ of $\mathcal{G}$ agrees with the group character induced on $\mathcal{G}$ from the trivial character $\chi = 1_H^{\mathcal{G}}$ of the subgroup H. For this permutation character we have the relation

$$1_H^{\mathcal{G}}(x) = \frac{|x^{\mathcal{G}} \cap H|\cdot|C_{\mathcal{G}}(x)|}{|H|} = \frac{|x^{\mathcal{G}} \cap H|\cdot|C_{\mathcal{G}}(x)|}{|x^H \cap H|\cdot|C_H(x)|}. \tag{1}$$

If $\{x^{\mathcal{G}} \cap H\} = \bigcup_{i=1}^{k}\{x_i^H \cap H\}$, this relation can be written in the form

$$1_H^{\mathcal{G}}(x) = \sum\nolimits_{i=1}^{k} \frac{|G_{\mathcal{G}}(x)|}{|C_H(x_i)|}, \tag{2}$$

where $x^{\mathcal{G}} = \{g^{-1}xg \mid g \in \mathcal{G}\}$ is the class of elements conjugate to x in $\mathcal{G}$ and $x_1, \ldots, x_k$ are representatives of the different conjugacy classes in H into which the set $\{x^{\mathcal{G}} \cap H\}$ splits. Obviously $\chi(x) = 1$ if and only if $|C_{\mathcal{G}}(x)| = |C_H(x)|$ and $x^{\mathcal{G}} \cap H = x^H \cap H$, and $\chi(x) = 0$ if and only if $x^{\mathcal{G}} \cap H = \varnothing$.

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