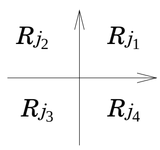
Program for computing Fiedler's (long) knot invariant W

S.Yu. Orevkov

May 16, 2007

The file `http://picard.ups-tlse.fr/~orevkov/fie-w` contains a program for 'Mathematica' which computes the invariant W of a long oriented knot constructed by Fiedler [1]. A long knot is presented in the following way. We fix a knot diagram (a generic projection to a plane from a point on the knot). To each crossing we associate its *writhe* (a sign) in the usual way. Let $R_1, \dots, R_m$ be the *regions* (i.e., the components) of the complement of the knot diagram (in fact, $m = n + 2$) indexed so that R_1 and R_2 are the unbounded regions (recall that the knot is long), and the order of the other regions is arbitrary.

For a crossing p, let us denote the regions adjacent to p by R_{j_1}, R_{j_2}, R_{j_3}, R_{j_4} counterclockwise:

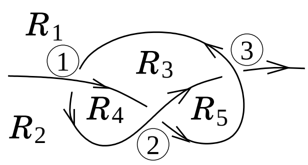


and let the *region-vector* of p be the vector $[j_1, j_2, j_3, j_4]$.

The Fiedler's invariant W is computed by the command `W[knot]` where `knot` is the list `{Wr, reg}`, `Wr` is the list of writhes, and `reg` is the list of region-vectors.

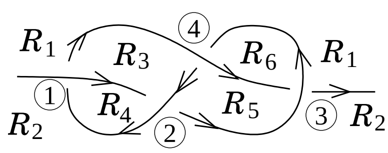
Example 1. Negative trefoil.

```
In[1]:= << fie-w
In[2]:= Wr3={-1,-1,-1};
In[3]:= reg3={{4, 3, 1, 2},
              {5, 3, 4, 2},
              {1, 3, 5, 2}};
In[4]:= W[{Wr3,reg3}]
```



Example 2. Figure-eight knot.

```
In[5]:= Wr8={1,1,-1,-1};
In[6]:= reg8={{3, 1, 2, 4},
              {2, 5, 3, 4},
              {1, 6, 5, 2},
              {5, 6, 1, 3}};
In[7]:= W[{Wr8,reg8}]
```



Known bug: the program `W` works incorrectly when there is a bounded region whose closure is not simply connected (for example, for diagrams obtained by the first Reidemeister move).

[1]. T. Fiedler. *A link polynomial via vertex-edge-face model.* `http://arxiv.org/abs/0704.2953`