An index theorem for manifolds with boundary

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Abstract

In [2] II.5, Connes gives a proof of the Atiyah-Singer index theorem for closed manifolds by using deformation groupoids and appropiate actions of these on $\mathbb{R}^N$. Following these ideas, we prove an index theorem for manifolds with boundary.

Résumé

Un théorème d'indice pour des variétés à bord. Dans [2] II.5, Connes donne une preuve du théorème de l'indice d'Atiyah-Singer pour des variétés fermées en utilisant des groupoïdes de déformation et des actions appropriées de ceux-ci dans $\mathbb{R}^N$. Nous suivons ces idées pour montrer un théorème d'indice pour des variétés à bord.

Version française abrégée

Dans [2], II.5, Alain Connes donna une preuve du théorème d'Atiyah-Singer pour une variété fermée entièrement fondée sur l'utilisation de groupoïdes, grâce à une action du groupoïde tangent de la variété sur $\mathbb{R}^N$. L'idée centrale est de remplacer des groupoïdes qui ne sont pas (Morita) équivalents à des espaces, par des groupoïdes obtenus par produit croisé et qui possèdent cette propriété, ce qui permet ensuite de donner une formule.

Si X est une variété à bord ∂X, nous construisons le groupoïde $\mathcal{T}_b X := ({}^{ad}G_{\partial X} \times \mathbb{R}) \bigcup_\partial TX$ en recollant ${}^{ad}G_{\partial X} \times \mathbb{R}$ avec TX le long de leur bord commun $T\partial X \times \mathbb{R}$ (ici ${}^{ad}G_{\partial X} = T\partial X \cup \partial X \times \partial X \times (0,1)$ est le groupoïde adiabatique). Nous le recollons alors avec le groupoïde tangent de l'intérieur de X, ${}^T G_{\overset{\circ}{X}} = T\overset{\circ}{X} \cup \overset{\circ}{X} \times \overset{\circ}{X} \times (0,1]$: ${}^T G_X := \mathcal{T}_b X \bigcup_0 {}^T G_{\overset{\circ}{X}}$.

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Il existe un homomorphisme ${}^TG_X \xrightarrow{h} \mathbb{R}^N$ induit par un plongement de X dans $\mathbb{R}^{N-1}\times\mathbb{R}_+$, tel que ∂X se plonge dans $\mathbb{R}^{N-1}\times\mathbb{R}_+\times\{0\}$ et $\overset{\circ}{X}$ se plonge dans $\mathbb{R}^{N-1}\times\mathbb{R}_+^*$. Le produit croisé de ${}^TG_X$ par h (noté ${}^T(G_X)_h$) est un groupoïde propre dont les groupes d'isotropie sont triviaux, il est donc Morita-équivalent à son espace d'orbites.

Soit $V(\overset{\circ}{X})$ le fibré normal de $\overset{\circ}{X}$ dans $\mathbb{R}^N$, et $V(\partial X)$ le fibré normal de ∂X dans $\mathbb{R}^{N-1}$; soit enfin $V(X)=V(\overset{\circ}{X})\bigcup V(\partial X)$. En notant $\mathscr{D}_\partial = V(\partial X)\times\{0\}\bigsqcup\mathbb{R}^{N-1}\times(0,1)$ et $\mathscr{D}_\circ = V(\overset{\circ}{X})\times\{0\}\bigsqcup\mathbb{R}^N\times(0,1]$ les déformations au cône normal, on construit les espaces $\mathscr{B}_\partial := V(X)\bigcup_\partial \mathscr{D}_\partial$ et $\mathscr{B} := \mathscr{B}_\partial\bigcup_\circ \mathscr{D}_\circ$.

Proposition 0.1 *Le groupoïde $({}^TG_X)_h$ est Morita équivalent à l'espace $\mathscr{B}$.*

Soit

$$ind_f^X = (e_1)_*\circ(e_0)_*^{-1} : K^0(\mathcal{T}_bX)\longrightarrow K^0(\overset{\circ}{X}\times\overset{\circ}{X})\approx\mathbb{Z}.$$

Définition 0.1 (Indice topologique pour une variété à bord) *Soit X une variété à bord. L'indice topologique de X est le morphisme*

$$ind_t^X : K^0(\mathcal{T}_bX)\longrightarrow\mathbb{Z}$$

défini comme la composition des trois morphismes suivants

(i) L'isomorphisme de Connes-Thom CT_0 suivi de l'équivalence de Morita $\mathscr{M}_0$:

$$K^0(\mathcal{T}_bX)\xrightarrow{CT_0}K^0((\mathcal{T}_bX)_{h_0})\xrightarrow{\mathscr{M}_0}K^0(\mathscr{B}_\partial),$$

où $(\mathcal{T}_bX)_{h_0}$ est le produit croisé de $\mathcal{T}_bX$ par h_0 (l'homomorphisme h en $t=0$).

(ii) Le morphisme indice de l'espace de déformation $\mathscr{B}$: $K^0(\mathscr{B}_\partial)\underset{\approx}{\xleftarrow{(e_0)_*}}K^0(\mathscr{B})\xrightarrow{(e_1)_*}K^0(\mathbb{R}^N)$

(iii) Le morphisme de périodicité de Bott : $K^0(\mathbb{R}^N)\xrightarrow{Bott}\mathbb{Z}$.

Theorem 0.2 *Pour toute variété à bord, on a l'égalité*

$$ind_f^X = ind_t^X.$$

1. Actions of $\mathbb{R}^N$

All the groupoids considered here will be continuous family groupoids [5,11]. Hence we can consider their convolution and C^*-algebras without any problem. If G is such a groupoid, we will denote by $K^0(G)$ the K-theory group of its C^*-algebra (unless explicetely written otherwise). This is consistent with the usual notation when G is a space (a groupoid made only of units). In the sequel, given a smooth manifold N, we will denote by ${}^{ad}G_N : TN\times\{0\}\bigsqcup N\times N\times\mathbb{R}^*\rightrightarrows N\times\mathbb{R}$, the deformation to normal cone of N in $N\times N$(for complete details about this deformation functor see [1]). At each time, we will need to restrict it to some interval, e.g. $[0,1]$ gives the tangent groupoid, and $[0,1)$ gives the adiabatic groupoid.

Let $G\rightrightarrows M$ be a groupoid, as classically, the notation says G is the space of arrows (or morphisms) and M is the space of units (or objects). Let $h : G\to\mathbb{R}^N$ be a (smooth or continuous) homomorphism of groupoids, ($\mathbb{R}^N$ as an additive group). Connes defined the semi-direct product groupoid $G_h = G\times\mathbb{R}^N\rightrightarrows M\times\mathbb{R}^N$ ([2], II.5) with structure maps $t(\gamma,X)=(t(\gamma),X)$, $s(\gamma,X)=(s(\gamma),X+h(\gamma))$ and product $(\gamma,X)\circ(\eta,X+h(\gamma))=(\gamma\circ\eta,X)$ for composable arrows.

At the level of C^*-algebras, $C^*(G_h)$ can be seen as the crossed product algebra $C^*(G)\rtimes\mathbb{R}^N$ where $\mathbb{R}^N$ acts on $C^*(G)$ by automorphisms by the formula: $\alpha_X(f)(\gamma)=e^{i\cdot(X\cdot h(\gamma))}f(\gamma)$, $\forall f\in C_c(G)$, (see [2], propostion II.5.7 for details). In particular, in the case N is even, we have a Connes-Thom isomorphism in K-theory $K^0(G)\overset{\approx}{\to}K^0(G_h)$ ([2], II.C).

Using this groupoid, Connes gives a conceptual, simple proof of the Atiyah-Singer Index theorem for closed smooth manifolds. Let M be a smooth manifold, $G_M = M \times M$ its groupoid, and consider the tangent groupoid ${}^T G_M$. It is well known that the index morphism provided by this deformation groupoid is precisely the analytic index of Atiyah-Singer, [2,9]. In other words, the analytic index of M is the morphism

$$K^0(TM) \xrightarrow{(e_0)_*^{-1}} K^0({}^T G_M) \xrightarrow{(e_1)_*} K^0(M \times M) = K^0(\mathscr{K}(L^2(M))) \approx \mathbb{Z}, \tag{1}$$

where e_t are the obvious evaluation algebra morphisms at t. As discussed by Connes, if the groupoids appearing in this interpretation of the index were equivalent to spaces then we would immediately have a geometric interpretation of the index. Now, $M \times M$ is equivalent to a point (hence to a space), but the other fundamental groupoid playing a role is not, that is, TM is a groupoid whose fibers are the groups $T_x M$, which are not equivalent (as groupoids) to a space. The idea of Connes is to use an appropriate action of the tangent groupoid in some $\mathbb{R}^N$ in order to translate the index (via a Thom isomorphism) in an index associated to a deformation groupoid which will be equivalent to some space.

2. Groupoids and Manifolds with boundary

Let X be a manifold with boundary ∂X. We denote, as usual, $\overset{\circ}{X}$ the interior which is a smooth manifold. Let X_∂ be the smooth manifold obtained by glueing X with $\partial X \times [0,1)$ along their common boundary, $\partial X \sim \partial X \times \{0\}$. Set $TX := TX_\partial|_X$, and consider the smooth manifold $\mathcal{T}_b X := ({}^{ad}G_{\partial X} \times \mathbb{R}) \bigcup_\partial TX$ obtained by glueing ${}^{ad}G_{\partial X} \times \mathbb{R}$ and TX along their common boundary $T\partial X \times \mathbb{R}$ (${}^{ad}G_{\partial X} = T\partial X \cup \partial X \times \partial X \times (0,1) \rightrightarrows \partial X \times [0,1)$ is the adiabatic groupoid). Now, we have a continuous family groupoid over X_∂: $\mathcal{T}_b X \rightrightarrows X_\partial$. As a groupoid it is the union of the groupoids ${}^{ad}G_{\partial X} \times \mathbb{R} \rightrightarrows \partial X \times [0,1)$ (where $\mathbb{R} \rightrightarrows \{0\}$ as additive group) and $TX \rightrightarrows X$. For the topology, it is very easy to see that all the groupoid structures are compatible with the glueings we considered.

We are going to consider a deformation groupoid ${}^T G_X$ ([10]). This will be a natural generalisation of the Connes tangent groupoid of a smooth manifold, to the case with boundary. The space of arrows ${}^T G_X := \mathcal{T}_b X \bigcup_\circ {}^T G_{\overset{\circ}{X}}$ is obtained by glueing at $T\overset{\circ}{X}$ ($T\overset{\circ}{X} \times \{0\} \subset {}^T G_{\overset{\circ}{X}}$ is closed). The space of units $X_{g_0} := X_\partial \bigcup_\circ \overset{\circ}{X} \times [0,1]$ is obtained by glueing $\overset{\circ}{X} \sim \overset{\circ}{X} \times \{0\}$ ($\overset{\circ}{X} \times \{0\} \subset \overset{\circ}{X} \times [0,1]$ is closed). Using the groupoid structures of $\mathcal{T}_b X \rightrightarrows X_\partial$ and of ${}^T G_{\overset{\circ}{X}} \rightrightarrows \overset{\circ}{X} \times [0,1]$, we have a continuous family groupoid ${}^T G_X \rightrightarrows X_{g_0}$. Again, all the groupoid structures are compatible with the considered glueings.

To define a homomorphism ${}^T G_X \xrightarrow{h} \mathbb{R}^N$ we will need as in the nonboundary case an appropiate embedding. It is possible to find an embedding $i : X \hookrightarrow \mathbb{R}^{N-1} \times \mathbb{R}_+$ such that its restrictions to the interior and to the boundary are (smooth embeddings) of the following form $i_\circ : \overset{\circ}{X} \hookrightarrow \mathbb{R}^{N-1} \times \mathbb{R}_+^*$ and $i_\partial : \partial X \hookrightarrow \mathbb{R}^{N-1} \times \{0\}$. We define the homomorphism $h : {}^T G_X \to \mathbb{R}^N$ as follows.

$$h : \begin{cases} h(x, X, 0) = d_x i_\circ(X) \text{ and } h(x, y, \epsilon) = \dfrac{i_\circ(x) - i_\circ(y)}{\epsilon} \text{ on } {}^T G_{\overset{\circ}{X}} \\ h(x, \xi, 0, \lambda) = (d_x i_\partial(\xi), \lambda) \text{ and } h(x, y, \epsilon, \lambda) = (\dfrac{i_\partial(x) - i_\partial(y)}{\epsilon}, \lambda) \text{ on } {}^T G_{\partial X} \times \mathbb{R} \\ h(x, X) = d_x i_\circ(X) \text{ on } T\overset{\circ}{X} \end{cases} \tag{2}$$

Since all these morphisms are compatible with the glueings, one has:

Proposition 2.1 *With the formulas defined above, $h : {}^T G_X \to \mathbb{R}^N$ defines a homomorphism of continuous family groupoids.*

The action of ${}^TG_X$ on $\mathbb{R}^N$ defined by h is free because i is an immersion. It is not necessarily proper (in the case of Connes [2] II.5 it is since M was supposed closed), however we can prove the following:

Proposition 2.2 *The groupoid $({}^TG_X)_h$ is a proper groupoid with trivial isotropy groups.*

Notice that the groupoid G_h is not the transformation groupoid of a group action (if not, the properness of the action would be equivalent to the properness of the groupoid). It can be seen however as a transformation groupoid of a groupoid action. It is very important that the units of the groupoid G_h be the units of G times $\mathbb{R}^N$.

As an immediate consequence of the propositions above, the groupoid $({}^TG_X)_h$ is Morita equivalent to its space of orbits (see [7] example 5.33). Let us specify this space.

Let $V(\overset{\circ}{X})$ be the total space of the normal bundle of $\overset{\circ}{X}$ in $\mathbb{R}^N$. Similarly, let $V(\partial X)$ be the total space of the normal bundle of ∂X in $\mathbb{R}^{N-1}$. Observe that they have the same fiber vector dimension. In fact, their union $V(X) = V(\overset{\circ}{X}) \bigcup V(\partial X)$, is a vector bundle over X, the normal bundle of X in $\mathbb{R}^N$.

Take $\mathscr{D}_\partial = V(\partial X)\times\{0\} \bigsqcup \mathbb{R}^{N-1}\times(0,1)$ the deformation to the normal cone associated to the embedding $\partial X \overset{i_\partial}{\hookrightarrow} \mathbb{R}^{N-1}$. We consider the space $\mathscr{B}_\partial := V(X) \bigcup_\partial \mathscr{D}_\partial$ glued over their common boundary $V(\partial X) \sim V(\partial X)\times\{0\}$. On the other hand, take $\mathscr{D}_\circ = V(\overset{\circ}{X}) \times \{0\} \bigsqcup \mathbb{R}^N \times (0,1]$ the deformation to the normal cone associated to the embedding $\overset{\circ}{X} \overset{i_\circ}{\hookrightarrow} \mathbb{R}^N$. We consider the space $\mathscr{B} := \mathscr{B}_\partial \bigcup_\circ \mathscr{D}_\circ$ glued over $V(\overset{\circ}{X})$ by the identity map.

Proposition 2.3 *The space of orbits of the groupoid $({}^TG_X)_h$ is $\mathscr{B}$.*

We can give the explicit homeomorphism. The orbit space of $({}^TG_X)_h$ is a quotient of $X_{g_0} \times \mathbb{R}^N$. To define a map $\Psi : X_{g_0} \times \mathbb{R}^N \to \mathscr{B}$ it is enough to define it for each component of X_{g_0}. Let

$$\Psi : \begin{cases} \partial X \times (0,1) \times \mathbb{R}^{N-1} \times \mathbb{R} \to \mathbb{R}^{N-1} \times (0,1) \\ \Psi(a,t,\xi,\lambda) := (\dfrac{i_\partial(a)}{t} + \xi, t) \end{cases} \quad \begin{cases} \partial X \times \{0\} \times \mathbb{R}^{N-1} \times \mathbb{R} \to V(\partial X) \\ \Psi(a,0,\xi,\lambda) := \overline{(i_\partial(a),\xi)} \end{cases} \tag{3}$$

$$\begin{cases} \overset{\circ}{X} \times (0,1] \times \mathbb{R}^N \to \mathbb{R}^N \times (0,1] \\ \Psi(x,t,X) := (\dfrac{i_\circ(x)}{t} + X, t) \end{cases} \quad \begin{cases} \overset{\circ}{X} \times \{0\} \times \mathbb{R}^N \to V(\overset{\circ}{X}) \\ \Psi(x,0,X) := \overline{(i_\circ(x), X)} \end{cases}$$

where $\overline{\xi}$ denotes the class in $V_a(\partial X) := \mathbb{R}^{N-1}/T_{i_\partial(a)}\partial X$ (resp. $\overline{X}$ denotes the class in $V_x(\overset{\circ}{X}) := \mathbb{R}^N/T_{i_\circ(x)}\overset{\circ}{X}$). This gives a continuous map $\Psi : X_{g_0} \times \mathbb{R}^N \to \mathscr{B}$ that passes to the quotient into a homeomorphism $\overline{\Psi}$: $(X_{g_0} \times \mathbb{R}^N)/\sim \to \mathscr{B}$, where $(X_{g_0} \times \mathbb{R}^N)/\sim$ is the orbit space of the groupoid $({}^TG_X)_h$.

There is an alternative interpretation for $\mathscr{B}$ (we thank the referee for this suggestion): take the embedding $i : X \hookrightarrow \mathbb{R}^{N-1} \times \mathbb{R}_+$ and an appropriate tubular neighborhood U in $\mathbb{R}^{N-1} \times \mathbb{R}_+$; then $\mathscr{B}$ is diffeomorphic to $U \bigcup \mathbb{R}^{N-1} \times \mathbb{R}_+^*$.

3. The index theorem for manifolds with boundary

Deformation groupoids induce index morphisms. The groupoid ${}^TG_X$ is naturally parametrized by the closed interval $[0,1]$. Its algebra comes equipped with evaluations to the algebra of $\mathcal{T}_bX$ (at t=0) and to the algebra of $\overset{\circ}{X} \times \overset{\circ}{X}$ (for $t \neq 0$). We have a short exact sequence of C^*-algebras

$$0 \longrightarrow C^*(\overset{\circ}{X} \times \overset{\circ}{X} \times (0,1]) \longrightarrow C^*({}^TG_X) \overset{e_0}{\longrightarrow} C^*(\mathcal{T}_bM) \longrightarrow 0 \tag{4}$$

where the algebra $C^*(\overset{\circ}{X} \times \overset{\circ}{X} \times (0,1])$ is contractible. Hence applying the K-theory functor to this sequence we obtain an index morphism

$$ind_f^X = (e_1)_* \circ (e_0)_*^{-1} : K^0(\mathcal{T}_bX) \longrightarrow K^0(\overset{\circ}{X} \times \overset{\circ}{X}) \approx \mathbb{Z}.$$

The morphism $h : {}^TG_X \to \mathbb{R}^N$ is by definition also parametrized by $[0,1]$, *i.e.*, we have morphisms $h_0 : \mathcal{T}_bX \to \mathbb{R}^N$ and $h_t : \overset{\circ}{X} \times \overset{\circ}{X} \to \mathbb{R}^N$, for $t \neq 0$. We can consider the associated groupoids, which satisfy the same properties as in proposition 2.2 (in fact, for proving such proposition it is better to do it for each t, and to check all the compatibilities).

Définition 3.1 *[Topological index morphism for a manifold with boundary] Let X be a manifold with boundary. The topological index morphism of X is the morphism*

$$ind_t^X : K^0(\mathcal{T}_bX) \longrightarrow \mathbb{Z}$$

defined (using an embedding as above) as the composition of the following three morphisms

(i) The Connes-Thom isomorphism CT_0 followed by the Morita equivalence $\mathscr{M}_0$:

$$K^0(\mathcal{T}_bX) \xrightarrow{CT_0} K^0((\mathcal{T}_bX)_{h_0}) \xrightarrow{\mathscr{M}_0} K^0(\mathscr{B}_\partial)$$

(ii) The index morphism of the deformation space $\mathscr{B}$: $K^0(\mathscr{B}_\partial) \underset{\approx}{\xleftarrow{(e_0)_*}} K^0(\mathscr{B}) \xrightarrow{(e_1)_*} K^0(\mathbb{R}^N)$

(iii) The usual Bott periodicity morphism: $K^0(\mathbb{R}^N) \xrightarrow{Bott} \mathbb{Z}$.

Remark 1 The topological index defined above is a natural generalisation of the topological index theorem defined by Atiyah-Singer. Indeed, in the boundaryless case, they coincide. The index of the deformation space $\mathscr{B}$ is quite easy to understand because we are dealing now with spaces (as groupoids the product is trivial), then the group $K^0(\mathscr{B})$ is the K-theory of the algebra of continuous functions vanishing at infinity $C_0(\mathscr{B})$ and the evaluation maps are completely explicit. In particular, if we identify $\mathscr{B}_\partial$ with an open subset of $\mathbb{R}^N$ (in the natural way), then the morphism (ii) above correspond to the canonical extension of functions of $C_0(\mathscr{B}_\partial)$ to $C_0(\mathbb{R}^N)$.

The following diagram, in which the morphisms CT and $\mathscr{M}$ are the Connes-Thom and Morita isomorphisms respectively, is trivially commutative:

$$\begin{array}{ccccc}
K^0(\mathcal{T}_bX) & \underset{\approx}{\xleftarrow{e_0}} & K^0({}^TG_X) & \xrightarrow{e_1} & K^0(\overset{\circ}{X} \times \overset{\circ}{X}) \\
CT \downarrow \approx & & CT \downarrow \approx & & CT \downarrow \approx \\
K^0((\mathcal{T}_bX)_{h_0}) & \underset{\approx}{\xleftarrow{e_0}} & K^0(({}^TG_X)_h) & \xrightarrow{e_1} & K^0((\overset{\circ}{X} \times \overset{\circ}{X})_{h_1}) \\
\mathscr{M} \downarrow \approx & & \mathscr{M} \downarrow \approx & & \mathscr{M} \downarrow \approx \\
K^0(\mathscr{B}_\partial) & \underset{\approx}{\xleftarrow{e_0}} & K^0(\mathscr{B}) & \xrightarrow{e_1} & K^0(\mathbb{R}^N),
\end{array} \quad (5)$$

The left vertical line gives the first part of the topological index map. The bottom line is the morphism induced by the deformation space $\mathscr{B}$. And the right vertical line is precisely the inverse of the Bott isomorphism $\mathbb{Z} = K^0(\{pt\}) \approx K^0(\overset{\circ}{X} \times \overset{\circ}{X}) \to K^0(\mathbb{R}^N)$. Since the top line gives ind_f^X, we obtain the following result:

Theorem 3.1 *For any manifold with boundary X, we have the equality of morphisms*

$$ind_f^X = ind_t^X.$$

The last result is intimately related with the main result of [4]. In fact, if we consider the consider the conic pseudomanifold naturally associated to X, the noncommutative spaces considered here are the same as the ones considered in ref.cit., which by the way appeared also in [3], for instance, $\mathcal{T}_bX$ is the "Poincaré dual" to the Conic pseudomanifold in ref.cit. In particular the analytic index of [4] coincide with our ind_f, and the main results are basically the same. The novelty in these notes is the use of Connes crossed products and the Connes-Thom morphisms instead of the Thom morphisms associated to deformation groupoids, and hence there is in principle a difference between the topological indices. As in the case of smooth manifolds, there should be a very closed relation between these two (Thom) approaches which we think is worth to analyse.

4. Perspectives

As discussed in [3,4,5], the index map ind_f^X computes the Fredholm index of a fully elliptic operator in the b-calculus of Melrose. The result proven here might be used to give a formula in relation to that of Atiyah-Patodi-Singer ([6]).

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